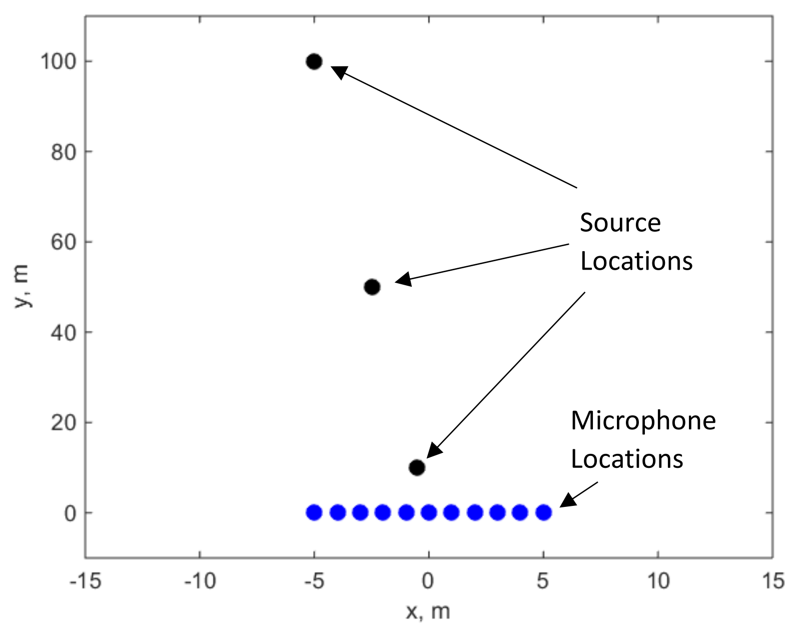
Chapter 12

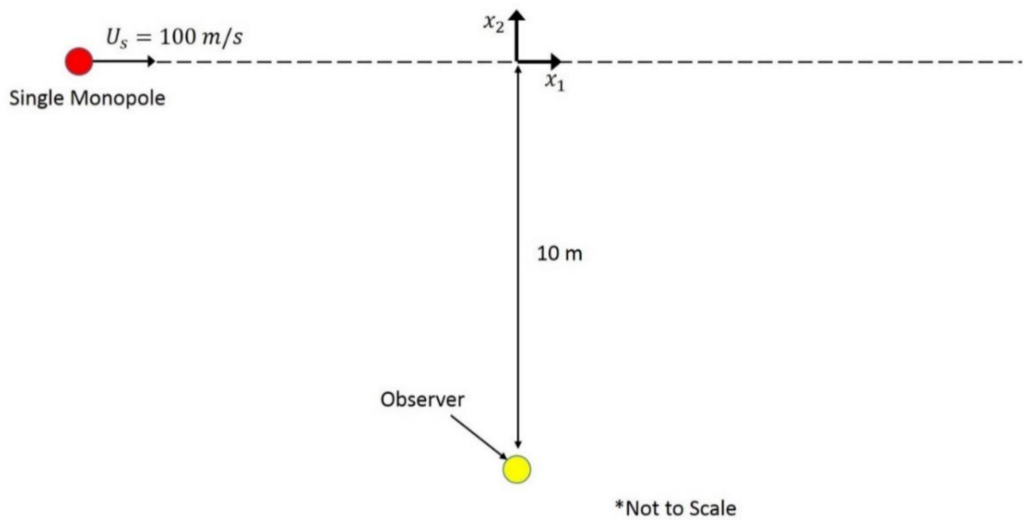
12.1 We commonly model isolated sources assuming a spherical spreading pattern as given by the equation for a monopole source, Equation 3.3.3. If a source is sufficiently far away from an array, the acoustic wavefronts may be assumed to be planar and the pressure at each microphone in an array can be approximated by Equation 12.1.2. Calculate and plot the variation in phase in degrees across microphones in the line array shown below assuming spherical waves emitting from a 1000 Hz source at locations progressively closer to the center of the array (coordinates given below). These are three separate cases. Do not consider the superposition of multiple sources in a single instance. The array has 11 sensors distributed with a spacing of 1 m in *x* along *y* = 0, *z* = 0 centered at the origin of the coordinate system. Calculate the phase variation in all cases relative to the phase of the central microphone of the array. Compare the calculated phase distributions for each case to the phase distribution assumed by plane waves approaching the array at the same angle of incidence relative to its center. Submit both your Matlab code and plot comparing the phase distribution for each source location and incident plane waves. Referencing these results, discuss the validity of the plane wave approximation for the various locations considering source frequency, as well.



Case	Source Coordinates, m
1	(-5, 100, 0)
2	(-2.5, 50, 0)
3	(-0.5, 10, 0)

Worked example solution

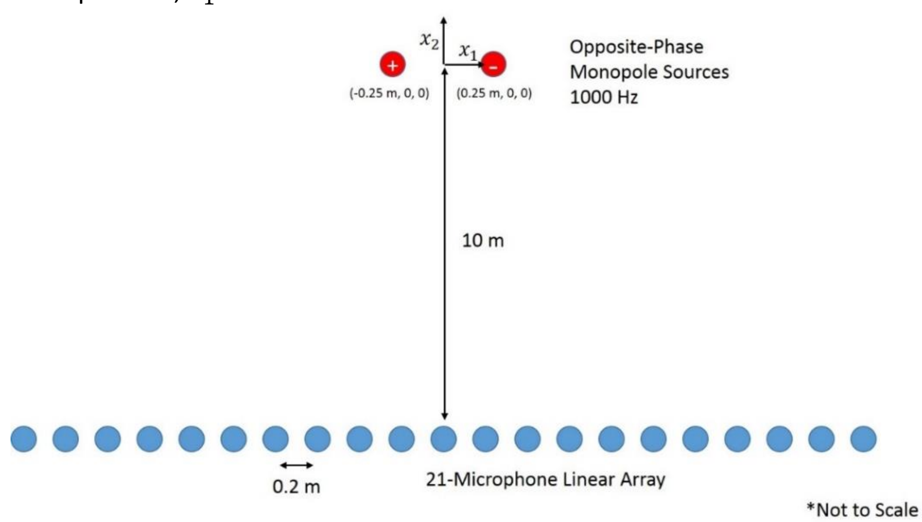
12.2 Consider a monopole source traveling at $U_s = 100$ m/s in the x_1 direction along a line 10 m from a stationary observer as shown in the figure below. The observer is located at (0, -10 m, 0).



(a) Calculate and plot the observer time as a function of retarded time as the monopole source travels from a location $x_1 = -$ 10 m to 10 m. Let $\tau = 0$ when the source is at $x_1 =$ -10 m.

(b) Explain in detail the origin of Doppler shift **and** Doppler amplification for a moving source. Qualitatively, how will these effects modify the sound heard by the observer in part (a) as the source passes by, when compared to the sound which would be heard if the source were stationary.

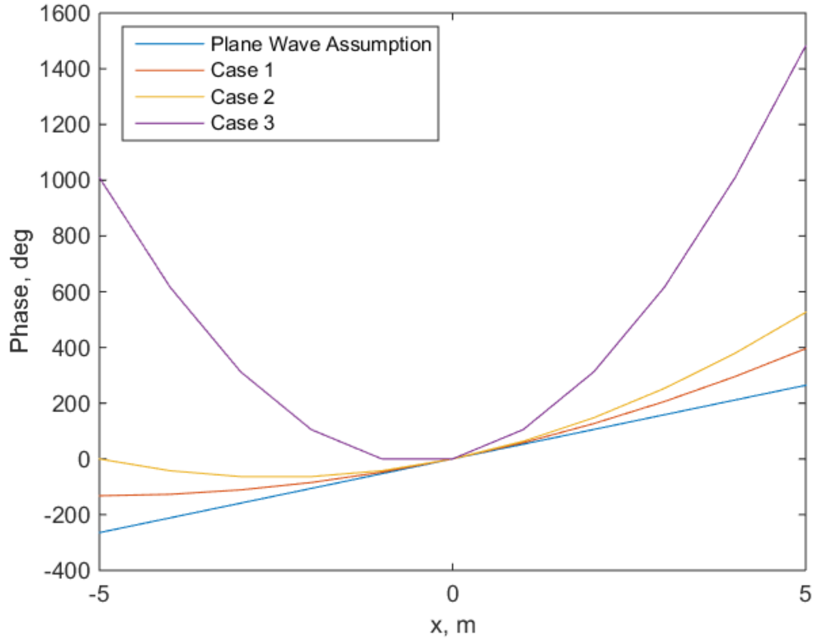
(c) Now consider a linear array of 21 microphones as shown below centered about $x_1 = 0$. There are two stationary monopole sources of equal strength and opposite phase located at (-0.25 m , 0, 0) and (0.25 m, 0, 0). These sources each produce a tone at 1 kHz. The elements of the array are spaced uniformly every 0.2 m in x_1. Plot the measured pressure amplitude and phase variation as a function of microphone position, x_1.



Using a spherical wave and monopole source assumption, plot the phase shift of the steering vector, ϕ_m, as a function of microphone position, x_1, using a focal point of (0.25 m, 0, 0) assuming the array is in the far field.

Solution Problem 12.1

```
clear all; close all;

%array coordinates
x1=[-5:5];
y1=zeros(1,11);

k=1000*2*pi/340; %acoustic wavenumber

sources=[-5 -2.5 -0.5;100 50 10];

for jj=1:size(sources,2)

    ro=sqrt(sources(1,jj).^2+sources(2,jj).^2);

    if jj==1

        %planar phase
        theta=atan(sources(1,jj)./sources(2,jj));
        phasePlanar=-k.*(([1:11]-1)-5).*sin(theta);

        figure(1)
        plot(x1,phasePlanar.*180/pi)
        hold on
    end

    %spherical phase
    r=sqrt((x1-sources(1,jj)).^2+(y1-sources(2,jj)).^2);
    phaseSpherical=k.*r-k.*ro;

    figure(1)
    plot(x1,phaseSpherical.*180/pi)
end

xlabel('x, m')
ylabel('Phase, deg')
legend('Plane Wave Assumption','Case 1','Case 2','Case 3')
```



The plane wave assumption requires that the array experience a phase change across its sensors consistent with that of an incident plane wave. In this case, the assumption improves with increasing source distance from the center of the array, but the phase error is still significant at the extents of the array even at the farthest distance studied here because of the high frequency. In general, the plane wave assumption improves if r_o, the distance to the source from the center of the array, increases such that it is much greater than the aperture of the array, L, but this appropriate distance is dependent upon frequency. The magnitude of the phase error using the plane wave assumption reduces at lower frequencies.