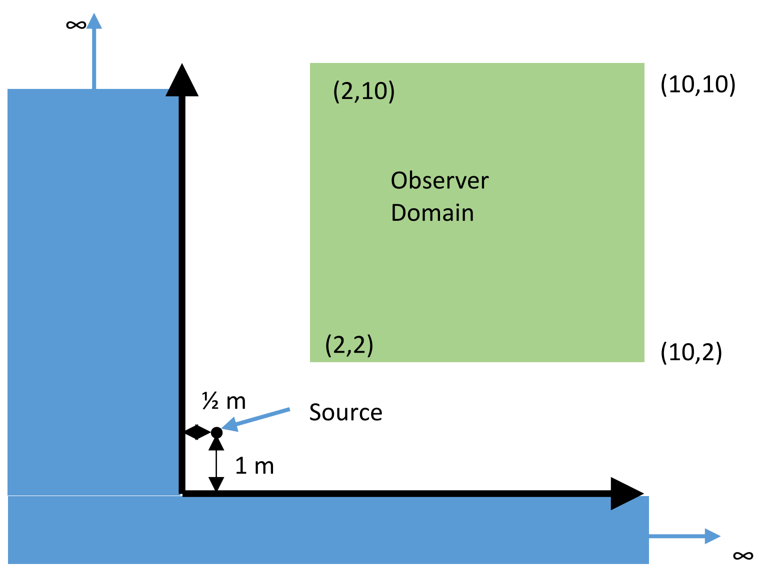
Chapter 4

4.1 A "Large Eddy Breakup Device" or LEBU is a small flat plate mounted a distance h above the wall in a turbulent boundary layer designed to disrupt passing eddies and change the characteristics of the boundary layer downstream. Because the flat plate acts as an airfoil, which responds to the varying angle of attack generated by the oncoming turbulence by producing an unsteady lift, then this device is also a noise producer.

a. Assuming the source strength (i.e. a harmonic unsteady force $F_2 = Re\{\hat{F}_2 e^{-i\omega\tau})$ is known and the LEBU is compact (but not necessarily its distance from the wall) write down an expression for acoustic pressure at a far field observer using a Green's function tailored to the boundary condition on the wall. You should assume that the flow is slow enough to have negligible effect on the propagation of sound.
b. A LEBU with a chord and span of 2cm is mounted in a 20cm thick turbulent boundary layer with an edge velocity of 15m/s at a distance of 10cm from the wall. Determine and plot the far-field root-mean-square acoustic pressure on a polar plot as a function of observer angle above the wall at a frequency of 5kHz if the flow is (i) in air, and (ii) in water. Normalize the far field RMS pressure on the RMS amplitude of the force on the LEBU and the observer distance from the point on the wall immediately below the LEBU. You may ignore the effects of convection on the sound field.

Worked example solution

4.2 Compute the tailored Green's function for a source at the location (1/2, 1, 0) m within an internal corner as shown below using the free field Green's function in the frequency domain given in Equation 3.10.8. Show the interference pattern produced by the corner for two different frequencies by making contour plots of the real part of the tailored Green's function as a function of observer positions over the domain x=([2:10],[2:10]) m. Assume a source in air with a frequency of 600 Hz and 60 Hz.

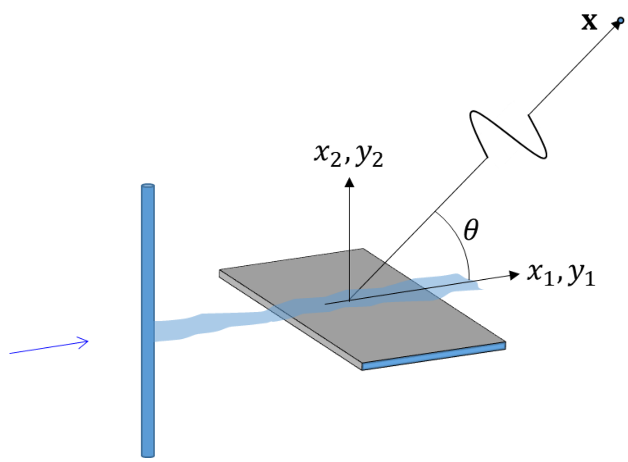


4.3 Estimate the noise made by a bumble bee in hover as heard by an observer 1m below the insect in terms of SPL. *Hint: It may help to research/estimate the weight of the bee and the frequency of its wing beats. Also, make use of Equation 4.4.7.*

4.4 A thin airfoil of chordlength c of 0.5m in an air flow of 30m/s sits downstream of a perpendicular strut that sheds vortices. As the vortices convect past the airfoil they generate an unsteady pressure distribution on the immersed portion of the span (which has a width of one tenth of the chord). The pressure distribution on the lower and upper surfaces of the airfoil (which may be idealized as a flat plate) has the form,

$$p_l(y_1,\tau) = A(\tau)\sqrt{\frac{a-y_1}{a+y_1}} \qquad\qquad p_u(y_1,\tau) = -A(\tau)\sqrt{\frac{a-y_1}{a+y_1}}$$

where the origin of the chordwise coordinate y_1 is at the mid chord, $a = c/2$, and $A(\tau) = Re\{Ce^{-i\omega\tau}\}$ where C is a constant. You may assume that the unsteady pressure is uniform and in phase across the immersed portion of the span. (Note that this fluctuating pressure distribution is more likely to be realistic at low frequencies than at high frequencies, but for the purposes of this question we will assume it applies equally at both.) Starting with Curle's equation (4.4.6) neglecting volume sources and viscous effects (likely negligible at the conditions given), $p'(\mathbf{x},t) = \frac{x_i}{4\pi|\mathbf{x}|^2 c_\infty}\int_S \left[\frac{\partial p}{\partial \tau}\right]_{\tau=\tau^*} dS(\mathbf{y})$,



(a) Find an analytical expression for the far field sound radiated by the airfoil for a frequency of 20Hz as a function of observer position $|\mathbf{x}|$ and θ in the plane perpendicular to the airfoil span. Introduce and justify appropriate simplifying assumptions as needed. See the notes below before attempting your solution.

(b) Analyze and compute the far-field sound for a frequency of 2000Hz. Specifically, compute and plot (in polar form) the mean square amplitude of the acoustic pressure, normalized on its maximum value, as a function of θ for $|\mathbf{x}| = 10\ m$. Be very careful about integration near the leading edge where the pressure distribution is singular (see notes below). Add a curve to compare your result with a compact dipole. Check your numerical method by adding a second plot for a frequency of 100 Hz case,that compares with the result from part (a). Make and justify appropriate simplifying assumptions as needed.

Notes

Please use, if needed, the following results:

i) $\int_{-1}^{1}\sqrt{\frac{1-s}{1+s}}\,ds = \pi$

ii) The mean square of $M\sin(\omega t) - N\cos(\omega t)$ in time is $\frac{1}{2}(M^2+N^2)$

iii) $\int_{-a}^{-a+\delta y_1} f(y_1)\sqrt{\frac{a-y_1}{a+y_1}}\,dy_1 \approx f(-a)\sqrt{2a}\int_{-a}^{a+\delta y_1}\sqrt{\frac{1}{a+y_1}}\,dy_1 = f(-a)2\sqrt{2a\delta y_1}$ where δy_1 is a small distance as long as f does not vary substantially between $-a$ and $-a+\delta y_1$

4.5 A large transport aircraft with a wing span of 60-m has landing gear wheels 1.1m in diameter, and a strut assembly 0.3-m in diameter. The principle source of noise from these elements is unsteady side force resulting from vortex shedding for which the fundamental frequency $fd/U \approx 0.2$, where U is the wind speed and d the component diameter.

(a) Determine whether, for a landing Mach number of 0.15, can we consider the wheels and strut assembly to be acoustically compact sources?

(b) In these two cases would a microphone mounted at the wing tip 30 m away be in the acoustic far field?

(c) The unsteady lift on the strut assembly (directed in the spanwise direction) has an RMS amplitude of $F_{rms}/0.5\rho_o U^2 S$ of 0.12, where $\rho_o = 1.1\ kg/m^3$ is the ambient air density and $S = 0.1\ m^2$ is the effective area of the strut assembly. If the direct line from the strut to the wing-tip microphone is at 60 degrees to the spanwise direction, estimate the amplitude of the acoustic pressure fluctuations at the microphone due to the strut assembly, ignoring scattering from any other parts of the aircraft. You may assume convection effects on the propagation of sound are negligible.

(d) Express your answer to part (c) in terms of acoustic intensity, giving the units.

Solution Problem 4.1

(a) Using a tailored Green's function we would start with the first term of equation 4.5.1

$$\boldsymbol{p}'(\mathbf{x},t)=\int_{-T}^{T}\int_{S} p'(\mathbf{y},\tau)\frac{\partial G_T}{\partial y_i}n_i dS(\mathbf{y})d\tau$$

Where, following equation 4.5.3,

$$G_T(\mathbf{x},t|\mathbf{y},\tau)=G_o(\mathbf{x},t|\mathbf{y},\tau)+G_o\left(\mathbf{x},t|\mathbf{y}^{\#},\tau\right)$$

And thus

$$\frac{\partial G_T(\mathbf{x},t|\mathbf{y},\tau)}{\partial y_i}=\frac{x_i-y_i}{r}\left(\frac{\dot{\delta}(t-r/c_o-\tau)}{4\pi r c_\infty}+\frac{\delta(t-r/c_o-\tau)}{4\pi r^2}\right)$$
$$+\frac{\partial y_i^{\#}}{\partial y_i}\frac{x_i-y_i^{\#}}{r}\left(\frac{\dot{\delta}(t-r^{\#}/c_o-\tau)}{4\pi r^{\#}c_\infty}+\frac{\delta(t-r^{\#}/c_o-\tau)}{4\pi {r^{\#}}^2}\right)$$

where $r=|\mathbf{x}-\mathbf{y}|$ and $r^{\#}=\left|\mathbf{x}-\mathbf{y}^{\#}\right|$ and $\frac{\partial y_i^{\#}}{\partial y_i}=(1,-1,1)$ with no summation implied. Substituting and integrating with respect to time and ignoring the near-field term we obtain,

$$\boldsymbol{p}'(\mathbf{x},t)=\int_S\left[\frac{x_i-y_i}{4\pi r^2 c_\infty}\frac{\partial p'}{\partial\tau}n_i\right]_{\tau=\tau^*}dS(\mathbf{y})+\int_S\left[\frac{\partial y_i^{\#}}{\partial y_i}\frac{x_i-y_i^{\#}}{4\pi {r^{\#}}^2 c_\infty}\frac{\partial p'}{\partial\tau}n_i\right]_{\tau=\tau^{*\#}}dS(\mathbf{y})$$

(summation now implied) where $\tau^*=t-|\mathbf{x}-\mathbf{y}|/c_\infty$ and $\tau^{*\#}=t-|\mathbf{x}-\mathbf{y}^{\#}|/c_\infty$. Now, $n_i=(0,1,0)$ on the top surface of the LEBU, where $y_2=-y_2^{\#}=h$ and $n_i=(0,-1,0)$ on the bottom surface (where $y_2=-y_2^{\#}=h$ also), then

$$p'(\mathbf{x},t)=\int_S\left[\frac{(x_2-h)}{4\pi r^2 c_\infty}\left(\frac{\partial p_u'}{\partial\tau}-\frac{\partial p_l'}{\partial\tau}\right)\right]_{\tau=\tau^*}dy_1dy_3-\int_S\left[\frac{(x_2+h)}{4\pi {r^{\#}}^2 c_\infty}\left(\frac{\partial p_u'}{\partial\tau}-\frac{\partial p_l'}{\partial\tau}\right)\right]_{\tau=\tau^{*\#}}dy_1dy_3$$

Since the integration here is only over the stationary compact surface of the LEBU, so variations of $r,r^{\#}$ and retarded time within the integrals are negligible, we can re-order the integrals as

$$p'(\mathbf{x},t)=\frac{(x_2-h)}{4\pi r_o^2 c_\infty}\left[\frac{\partial F_2}{\partial\tau}\right]_{\tau=\tau^*}-\frac{(x_2+h)}{4\pi {r_o^{\#}}^2 c_\infty}\left[\frac{\partial F_2}{\partial\tau}\right]_{\tau=\tau^{*\#}}$$

where r_o is distance from the center of the LEBU, and $r_o^{\#}$ distance from the center of its image in the wall. Now, $\left[\frac{\partial F_2}{\partial\tau}\right]_{\tau=\tau^*}=Re\left\{-i\omega\hat{F}_2 e^{-i\omega\left(t-\frac{r_o}{c_\infty}\right)}\right\}$, and likewise, $\left[\frac{\partial F_2}{\partial\tau}\right]_{\tau=\tau^{*\#}}=Re\left\{-i\omega\hat{F}_2 e^{-i\omega\left(t-\frac{r_o^{\#}}{c_\infty}\right)}\right\}$

So,

$$p'(\mathbf{x},t)=-\frac{(x_2-h)}{4\pi r_o^2 c_\infty}Re\left\{i\omega\hat{F}_2 e^{-i\omega\left(t-\frac{r_o}{c_\infty}\right)}\right\}+\frac{(x_2+h)}{4\pi {r_o^{\#}}^2 c_\infty}Re\left\{i\omega\hat{F}_2 e^{-i\omega\left(t-\frac{r_o^{\#}}{c_\infty}\right)}\right\}$$

$$p'(\mathbf{x},t) = Re\left\{\frac{ik\hat{F}_2 e^{-i\omega t}}{4\pi}\left(\frac{(x_2+h)e^{ikr_o^\#}}{r_o^{\#2}} - \frac{(x_2-h)e^{ikr_o}}{r_o^2}\right)\right\}$$

(b) The preceding equation is easily re-written as

$$p_{rms}(\mathbf{x}) = \frac{kF_{2rms}}{4\pi}\left|\frac{(x_2+h)e^{ikr_o^\#}}{r_o^{\#2}} - \frac{(x_2-h)e^{ikr_o}}{r_o^2}\right|$$

Or, in normalized form

$$\frac{p_{rms}(\mathbf{x})|\mathbf{x}|^2}{F_{2rms}} = \frac{k}{4\pi}\left|\frac{(x_2+h)e^{ikr_o^\#}}{r_o^{\#2}} - \frac{(x_2-h)e^{ikr_o}}{r_o^2}\right|$$

This function is evaluated and plotted for $|\mathbf{x}| = 100\ m$, $h = 0.1\ m$ for a frequency of 5kHz and $c_\infty = 340\ m/s$ (air) and $1500\ m/s$ (water) below.

```
clear all;close all;
th=[0:180]*pi/180;
h=0.1;x=100;x2=x*sin(th);
ro=sqrt(x^2*cos(th).^2+(x2-h).^2);
roh=sqrt(x^2*cos(th).^2+(x2+h).^2);

figure;subplot(1,2,1);
c=340;k=2*pi*5000/c; %Part (a) air flow
prms=k/4/pi*abs((x2+h).*exp(i*k*roh)./roh.^2-(x2-h).*exp(i*k*ro)./ro.^2);
polar(th,prms);
title('(a) Air flow');

subplot(1,2,2);
c=1500;k=2*pi*5000/c; %Part (b) water flow
prms=k/4/pi*abs((x2+h).*exp(i*k*roh)./roh.^2-(x2-h).*exp(i*k*ro)./ro.^2);
polar(th,prms);
title('(b) Water flow');
set(gcf,'Position',[0   351   736   351]);
```